Between Square and Hexagon in Oresme's *Livre du Ciel et du Monde*

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There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.

— William Shakespeare, *Hamlet*

1 Introduction

Aristotelian diagrams visually represent the elements of some logical, lexical or conceptual field, and the relations of contradiction, contrariety, subcontrariety and subalternation holding between them. These diagrams are widely studied and used in philosophical logic and linguistics. These two disciplines offer highly complementary perspectives on Aristotelian diagrams. On the one hand, logicians almost always assume that the diagrams are closed under negation: if such a diagram contains φ, then it also contains $\neg\varphi$ (up to logical equivalence) (Demey & Smessaert, 2017a; Smessaert & Demey, 2014). Consequently, the Aristotelian diagrams studied in logic are highly regular and *symmetric* in nature.[1] On the other hand, linguists often focus on those concepts that are (primitively) lexicalized in natural language (e.g. English), i.e. concepts that can be expressed by means of a single word. The property of being lexicalized is notoriously not closed under negation: it is possible for a notion to be lexicalized, while its negation is not lexicalized (Horn, 1989; Seuren & Jaspers, 2014). Consequently, the Aristotelian diagrams studied in linguistics are often more *asymmetric* in nature.

[1] This symmetry can mathematically be described in various ways, using tools from Euclidean geometry, group theory and graph theory (Demey & Smessaert, 2014, 2016b, 2017b; Smessaert & Demey, 2016).

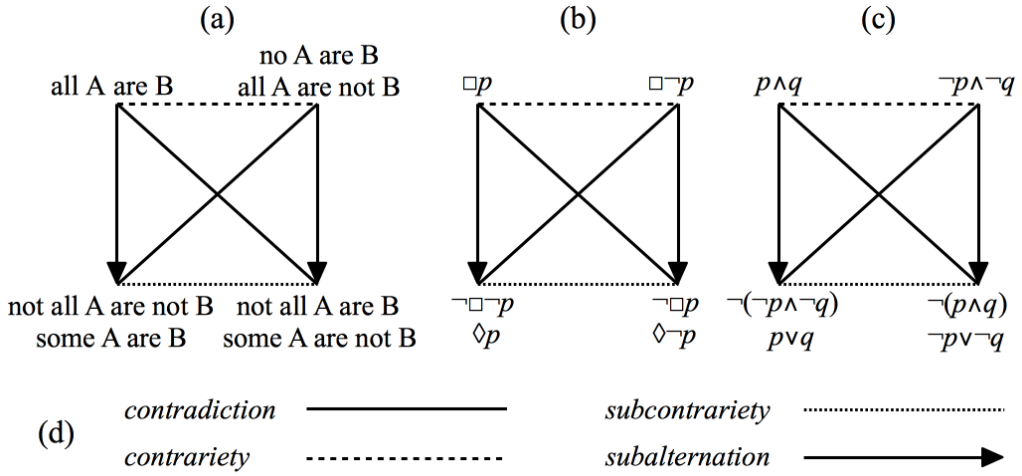

Figure 1: Squares of opposition for (a) syllogistics, (b) modal logic, (c) propositional logic; (d) code for visualizing the Aristotelian relations.

Dany Jaspers is a linguist with a keen interest in philosophical and logical issues. It should thus not be surprising that the intricate interplay between logical symmetry and linguistic asymmetry surfaces whenever Dany and I find ourselves discussing some aspect of Aristotelian diagrams. It often gives rise to an interesting dialectic, in which both perspectives can benefit and learn from each other, while mutually respecting the proper characteristics of one another. In this paper I will describe a particular example of this dialectic, and, most importantly, trace some of its historical roots.

2 From the Square to the Hexagon

Without a doubt, the oldest and most well-known Aristotelian diagram is the *square of opposition* (Parsons, 2017). Figure 1 shows squares of opposition for logical systems such as syllogistics, modal logic, and propositional logic. I will make use of the well-established vowel code (A/E/I/O) for labeling the four corners of the square, as shown in Figure 2(a). This is a typical example of an Aristotelian diagram as studied in logic, with a high degree of symmetry. It is closed under negation: the negation of I is E (and vice versa), and the negation of A is O (and vice versa).

From a linguistic perspective, there is a significant difference between the A-, I- and E-corners of the square on the one hand, and the O-corner on the other. Typically, A, I and E are lexicalized in natural language. For example, with the

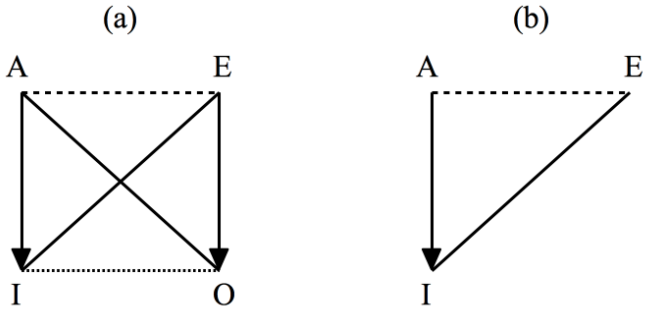

Figure 2: (a) Abstract square of opposition; (b) the lexicalized part of the square.

quantifiers we have *all*, *some* and *no*; with the modalities we have *necessary*, *possible* and *impossible*; and with the propositional connectives we have *and*, *or* and *nor*. By contrast, O is not lexicalized: there do not exist words such as **nall*, **nnecessary* and **nand*. This is known as the problem of the non-lexicalization of the O-corner (Horn, 1989, 2012; Katzir & Singh, 2013). In terms of diagrams, it means that we disregard the O-corner, and focus on the A-, I- and E-corners. This yields a *triangle of opposition*, as shown in Figure 2(b). Note that this triangle is not closed under negation: it contains A, but it does not contain the negation of A (viz. O).

Let us now switch back to the logical perspective. The square of opposition is closed under the Boolean operation of negation. Hence, a natural question to ask is whether it is also closed under the other Boolean operations of conjunction and disjunction.[2] This is not the case: (i) the square contains I and O, but it does not contain (a proposition that is logically equivalent to) their conjunction I $\wedge$ O (which is often labeled 'Y'); similarly, (ii) the square contains A and E, but it does not contain (a proposition that is logically equivalent to) their disjunction A $\vee$ E (which is often labeled 'U'). It can be shown that these are the only two Boolean combinations that are missing from the square. We can add them to the square, thereby obtaining a *hexagon of opposition*, as shown in Figure 3(a). This hexagon is the Boolean closure of the square: it is the smallest Aristotelian diagram that (i) is closed under all Boolean operations and (ii) contains the square as a subdiagram. It was first studied in the 1950s by Jacoby (1950), Sesmat (1951) and Blanché (1953), and is therefore nowadays called a 'Jacoby-Sesmat-Blanché (JSB) hexagon'. Together with Pieter Seuren, Dany Jaspers has done important work in charting the historical background to this crucial logical development

[2] Given the interdefinability of conjunction and disjunction (in the presence of negation), it follows that (i) if a diagram is closed under negation and conjunction, it is closed under disjunction as well, and (ii) if a diagram is closed under negation and disjunction, it is closed under conjunction as well.

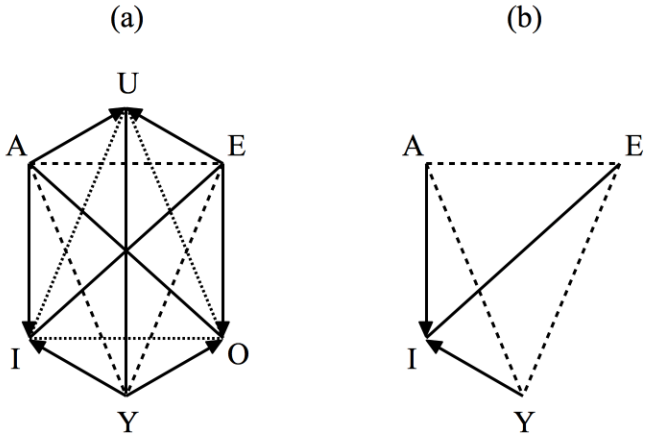

Figure 3: (a) Abstract JSB hexagon; (b) the lexicalized part of the JSB hexagon.

(Jaspers & Seuren, 2016).

From a linguistic perspective, there is again a significant difference between the Y- and the U-corner of the JSB hexagon. Typically, Y is lexicalized in natural language (often in the same way as I). For example, with the quantifiers we have (bilateral) *some*; with the modalities we have (bilateral) *possible*; and with the propositional connectives we have (exclusive) *or*. By contrast, U is not lexicalized: there do not exist words such as **allorno*, **necessaryorimpossible* and **andornor*. Ideally, we would want to have a theory that can simultaneously explain the non-lexicalization of the O-corner and that of the U-corner. Pieter Seuren and Dany Jaspers (Seuren & Jaspers, 2014) have developed precisely such a theory. In terms of diagrams, the linguistic perspective means that we disregard the O- and U-corners, and focus on the A-, I-, E- and Y-corners. This yields a *kite*, as shown in Figure 3(b). Note, again, that this kite is not closed under negation: it contains A and Y, but it does not contain the negation of A (viz. O) or the negation of Y (viz. U).[3]

3 Between Square and Hexagon

Unfortunately, logical and intellectual developments hardly ever proceed in the clear-cut and smooth fashion that I have described above. For example, one might want to add the Y-corner to the square (because it is lexicalized and/or rep-

[3]Another linguistically motivated, and thus asymmetric (i.e. not closed under negation), diagram that should be mentioned in this respect, is the so-called 'Jespersen triangle', which consists of A, E and Y (Jespersen 1917, 1924; also see Horn 2012, 398).

resents an interesting philosophical notion), *without* having to add the U-corner as well (because that one is not lexicalized and/or does not represent an interesting philosophical notion). In this way one obtains a *pentagon of opposition*, as shown in Figure 4(a); also see Horn (Horn, 2012, 403). This pentagon occupies an uneasy position 'between' the square of opposition and the JSB hexagon. On the one hand, it does not fit well with the logical perspective (because it is not closed under negation: it contains Y, but not the negation of Y), and on the other hand, it does not fit well with the linguistic perspective either (because it contains a notion that is typically not lexicalized, viz. O). Diagrammatically speaking, the pentagon can be seen as the result of superimposing the square of opposition and the kite: the former is a 'logic-oriented', symmetric diagram, whereas the latter is a 'linguistics-oriented', asymmetric diagram.[4]

Unnatural though it may be, concrete instances of the pentagon can effectively be found in the extant literature. For example, in a short article in 1970, Hilail Gildin (Gildin, 1970) used it to analyze Aristotle's moral theory; see Figure 4(b).[5] In this pentagon, A represents the vice of *rashness* (always standing up to danger) and E represents the vice of *cowardice* (never standing up to danger). Correspondingly, I stands for sometimes standing up to danger, while O stands for sometimes not standing up to danger. However, the true virtue, *courage*, occupies a 'middle ground' between the two vices, and is thus represented by the Y-corner: sometimes standing up to danger and sometimes not standing up to danger.

4 Oresme's *Livre du Ciel et du Monde*

The concrete example (Gildin, 1970) of a pentagon that I have just given is fairly recent (1970). In this section, however, I will discuss what is — to the best of my knowledge — the oldest example of a pentagon in the literature. This example is due to the 14th-century author Nicole Oresme, one of the most eminent scholastic philosophers, mathematicians and scientists (Kirschner, 2017).

Oresme lived in France from around 1320 to 1382 (Burton, 2007). His early career was spent at the University of Paris (Courtenay, 2000). From 1362 until his death, he served Charles, the dauphin of France, who was crowned King

[4] Alternatively, the pentagon can also be seen as the result of superimposing the square of opposition and the Jespersen triangle (cf. Footnote **??**); again, the former is a 'logic-oriented', symmetric diagram, whereas the latter is a 'linguistics-oriented', asymmetric diagram.

[5] Interestingly, Gildin himself called his diagram a 'square of opposition' rather than a pentagon — perhaps because of the popularity of the square, in contrast to the obscurity of the pentagon. Nevertheless, the diagram does contain a Y-corner, and should thus be seen as being first and foremost a *pentagon*.

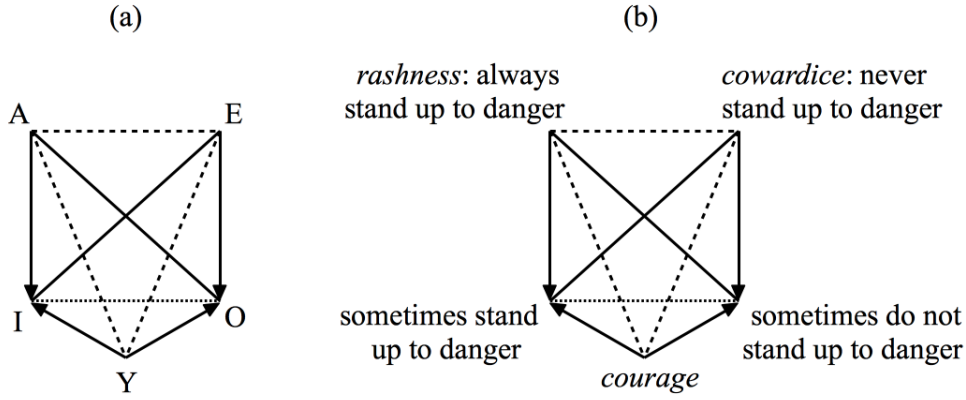

Figure 4: (a) Abstract pentagon of opposition; (b) Gildin's moral pentagon.

Charles V in 1364. Oresme was tasked by Charles to produce French translations of, and commentaries on, several of Aristotle's works (Kirschner, 2017).[6] One of these works was Aristotle's cosmological treatise *On the Heavens* (Περὶ οὐρανοῦ). This treatise was translated from Greek into Latin (as *De Caelo et Mundo*), first by Gerard of Cremona (1170), and later by Robert of Lincoln and William of Moerbeke (1250–1265).[7] Oresme used the latter as the basis for his translation from Latin into French (as *Le livre du ciel et du monde*) (Menut & Denomy, 1968). The first printed version of Oresme's translation appeared in the journal *Mediaeval Studies* from 1941 to 1943 (Menut & Denomy, 1941, 1942, 1943); a revised version, in which the Middle French text is accompanied by a contemporary English translation, was published in 1968 (Menut & Denomy, 1968).

In his *Livre du ciel et du monde*, Oresme discusses Aristotle's remarks regarding objects that have/do not have a beginning and objects that have/do not have an end, and he illustrates rates his discussion by means of a pentagon. He explicitly recognizes the similarity between his diagram and the more common square of opposition (Menut & Denomy, 1968, 220–221):

> "In order to illustrate this, I clarify it by means of a figure very similar to that used to introduce children to logic."

[6] From a historical-linguistic perspective, it is interesting to note that through these translations, Oresme had a considerable influence on the development of scientific and philosophical vocabulary in medieval French (Kirschner, 2017; Menut & Denomy, 1968).

[7] The Dominican William of Moerbeke (1215–1286) was one of the most prolific translators of philosophical and scientific treatises from Greek into Latin. He maintained a vivid correspondence with many prominent authors of his time, such as his fellow Dominican, Thomas Aquinas (1225–1274) (Brams & Vanhamel, 1989). As is suggested by his name, William originates from the Flemish town of Moerbeke, near Geraardsbergen.

(*Et pour ce mieux entendre, je le desclaire en une figure presque semblable a une que l'en fait pour la premiere introducion des enfans en logique.*)

In Oresme's pentagon, the A-corner stands for 'always possible to be' (*tousjours possible estre*), the E-corner stands for 'always possible not to be' (*tousjours possible non estre*), the I-corner stands for 'not always possible not to be' (*non pas tousjours possible non estre*) and the O-corner stands for 'not always possible to be' (*non pas tousjours possible estre*).[8] Finally, the Y-corner is simply labeled 'the intermediate' (*le moien*), but from the ensuing text it is clear that Oresme takes this to mean I $\wedge$ O (Menut & Denomy, 1968, 220–221):

> "Therefore, it is necessary that the two negations of the two [contraries][9], —that is, the two subcontraries— be said of the same identical thing and that this thing should be intermediate between always being and always not being. It is what is capable of being and of not being, for *each of the two subcontraries will sometimes be true*, but not always so." (emphasis added)
>
> (*Et pour ce convient par necessité que les negacions des .ii. [contraires], c'est assavoir les .ii. subcontraires, soient dictes d'une meisme chose et que celle chose soit moienne entre touzjours estre et touzjours non-estre. Et est la chose qui est possible estre et possible non-estre, quar chascune des .ii. negacions, qui sont subcontraires, sera vraie aucune foys, pousé que ce ne soit pas touzjours.*)

We conclude this section by taking a look at Oresme's actual pentagon, as it appears in one of the six remaining manuscripts of the *Livre du ciel et du monde*, viz. Ms. franç. 1082 of the Bibliothèque nationale de France (BnF), which is freely available online in digital format (Oresme, s.d.), and which constitutes the main source for the edition and translation in Menut & Denomy (1941, 1942, 1943, 1968). On fol. 51r of this manuscript, we find the diagram that is reproduced

[8] Note that by themselves, these A-, E-, I- and O-corners not only constitute an *Aristotelian square* (with relations of contradiction, (sub)contrariety and subalternation), but also a *duality square* (with relations of internal negation, external negation and duality) (Demey & Smessaert, 2016a). Furthermore, since the operator generating this square is itself the result of composing a temporal operator (*tousjours*) with a modal operator (*possible*), the square can naturally be extended to a duality *cube* (Demey, 2012).

[9] In the English translation in (Menut & Denomy, 1968, 221) we read 'contradictories' at this place, but this is clearly incorrect. In most manuscripts, the original French text simply has 'con', which is erroneously conjectured to stand for 'con⟨tradictoires⟩' (Menut & Denomy, 1968, 220). Furthermore, one of the remaining manuscripts of Oresme's *Livre* explicitly has 'contraires' here, which is at it should be (Menut & Denomy, 1968, 220, Footnote 2).

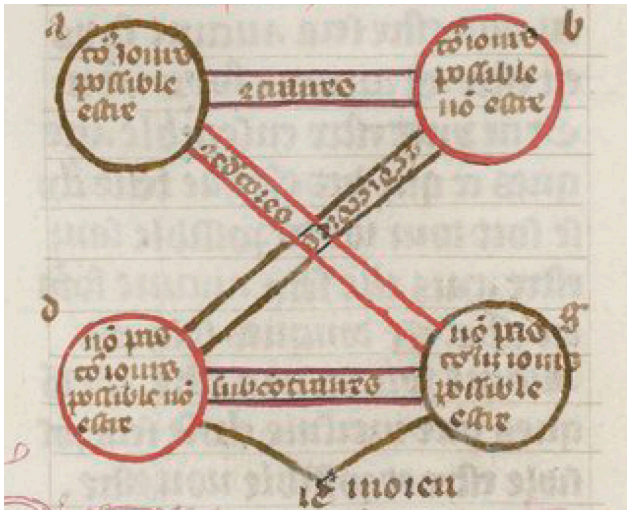

Figure 5: The pentagon of opposition in BnF Ms. franç. 1082, fol. 51r.

here as Figure 5. Note that the diagram is not complete; for example, the subalternations from A to I and from E to O are m issing, as well as the contrarieties between A and Y and between E and Y. Nevertheless, the key aspect of the pentagon, viz. the addition of a fifth corner (Y, *le moien*), is clearly discernible, thus making Oresme the earliest known author to have used this unusual type of Aristotelian diagram.

5 Conclusion

In this paper I have examined a strange Aristotelian diagram, viz. the pentagon of opposition. This pentagon occupies an uneasy position between the more common square of opposition and JSB hexagon. It also transcends the boundaries between the logical perspective (with its focus on closure under negation and the resulting symmetry) and the linguistic perspective (with its focus on lexicalization and the resulting asymmetry) on Aristotelian diagram, and should thus certainly appeal to a logic-minded linguist such as Dany Jaspers. I have also discussed the earliest known occurrence of the pentagon, which can be found in Nicole Oresme's late 14th-century *Livre du ciel et du monde*. The fact that an odd diagram such as the pentagon is found, not in an ordinary philosophy textbook but rather in a treatise on heaven and earth, clearly illustrates the truth of Hamlet's reply to Horatio.

Acknowledgements

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